Capítulo 3

Modelling and Modal Control of a Quarter Vehicle Suspension System

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Abstract: In this paper is presented the mathematical model of a quarter vehicle suspension system, it includes disturbances caused by the terrain irregularities. The work shows a model for a passive suspension and the model of a semiactive suspension system, which includes the damping force of a magnetorheological damper connected to the vehicle chassis. The MR damper has the function to attenuate the vertical movement, by means of the application of an algorithm of control. To minimize the chassis movement, is considered a modal control scheme based on the Positive Velocity Feedback (PVF). Finally some numerical results are included to show the dynamic perfomance of the passive suspension system and the response of the semiactive suspension system based on a Magnetorheological damper.

Keywords: Semiactive vehicle suspension, Magnetorheological damper, Modal Control, Vibrations absorption.

Resumen: En este artículo se presenta el modelo matemático de un sistema de suspensión de un cuarto de automóvil, el modelo incluye perturbaciones provocadas por las irregularidades del terreno. En el trabajo se muestra el modelo de un sistema de suspensión pasiva y el modelo de un sistema de suspensión semiactiva, el cual incluye la fuerza de un amortiguador magnetoreológico (MR) acoplado al chasis del vehículo. El amortiguador MR tiene la función de frenar el

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movimiento vertical en el chasis, mediante la aplicación de un algoritmo de control. Para minimizar el movimiento vertical en el chasis, se considera un esquema de control modal basado en la retroalimentación positiva de la velocidad. Finalmente se incluyen algunos resultados numéricos para mostrar el desempeño dinámico de un sistema de suspensión pasivo y la respuesta de un sistema de suspensión semiactivo basado en un amortiguador magnetoreológico.

Palabras clave: Suspensión semiactiva de vehículo, Amortiguador magnetorreológico, Control Modal, Absorción de vibraciones.

3.1 Introduction

The main control objectives of active vehicle suspension systems are to improve the ride comfort and handling performance of the vehicle by adding degrees of freedom to the passive system and/or controlling actuator forces depending on feedback and feedforward information of the system obtained from sensors. To do this, an actuator is incorporated to the suspension system to apply control forces on the body of the automobile to reduce its vertical movement in active/semiactive way.

Passenger comfort is provided by isolating the passengers from the undesirable vibrations induced by irregular road disturbances and its performance is evaluated by the level of acceleration by which vehicle passengers are exposed. Handling performance is achieved by maintaining a good contact between the tire and the road to provide guidance along the track.

The topic of active vehicle suspension control system, for linear and nonlinear models, in general, has been quite challenging over the years and we refer the reader to some of the fundamental works in the vibration control area in Ahmadian [Ahmadian2001]. Some active control schemes are based on neural networks, genetic algorithms, fuzzy logic, sliding modes, H-infinity, adaptive control, disturbance observers, LQR, backstepping control techniques, etc. See, e.g., Cao [Cao2008], Isermann [Isermann2011], Martins [Martins2006], Tahboub [Tahboub2005], Beltran *et al.* [Beltran2011], Chen [Chen2005] and references therein.

In addition, some interesting semiactive vibration control schemes, based on Electro-Rheological (ER) and Magneto-Rheological (MR) dampers, have been proposed and implemented on commercial vehicles. See, e.g., [Yao2002], [Choi2001b], [Guozhi2000].

The aplication of a modal control is shown to attenuate the vertical movement of the chassis in a suspension system based on the use of a magnetorheological (MR) damper and the application of Positive Velocity Feedback (PVF) control scheme (see Inman [Inman2006a], [Inman2006b] and references therein). The dynamics associated to the MR damper force is modeled using the Choi-Lee-Park polynomial model (Choi *et al.* [Choi2001a], Spencer *et al.* [Spencer1997]).

Some numerical simulation results are provided to show the efficiency, effectiveness and robust performance of the feedforward and feedback linearization control scheme proposed for a quarter-vehicle semiactive suspension system.

3.2 Linear mathematical model of a quarter vehicle suspension

An schematic diagram of a quarter car suspension system is shown in Figure 3.1.

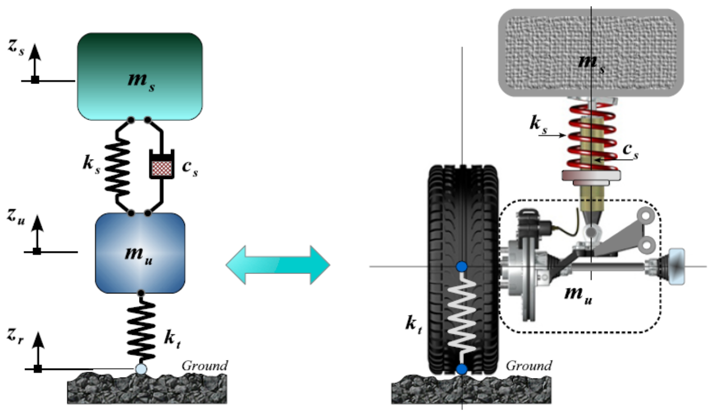

Figura 3.1: A quarter car passive suspension.

The mathematical model of passive suspension system is given by:

$$\begin{aligned} m_s\ddot{z}_s + c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0 \\ m_u\ddot{z}_u - c_s(\dot{z}_s - \dot{z}_u) - k_s(z_s - z_u) + k_t(z_u - z_r) = 0 \end{aligned} \tag{3.2.1}$$

The general model for the passive system is rewritten by:

$$M\ddot{q} + C\dot{q} + Kq = B_f u + B_r z_r$$

$$M = \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}, C = \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix}, K = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix}$$

$$B_r = \begin{bmatrix} 0 & k_t \end{bmatrix}^T, B_f = \begin{bmatrix} 0 & 0 \end{bmatrix}^T, \ q = \begin{bmatrix} z_s & z_u \end{bmatrix}$$

where m_s represents the masses of a quarter car, m_u represents the mass of one wheel with the suspension and brake equipment, c_s is the damper coefficient of suspension, k_s and k_t are the spring coefficients of suspension and the tire, z_s and z_u are the displacements of car body and wheel and z_r is the terrain input disturbance.

3.3 The suspension model with a MR damper

An schematic diagram of a quarter car passive suspension is shown in Fig. 3.1. The Magnetorheological Damper (MR damper) replaces the passive damper c_s, forming a suspension with the spring.

The mathematical model of a suspension with Magnetorheological Damper system is given by:

$$\begin{aligned} m_s\ddot{z}_s + k_s(z_s - z_u) + u = 0 \\ m_u\ddot{z}_u - k_s(z_s - z_u) + k_t(z_u - z_r) - u = 0 \end{aligned} \tag{3.3.1}$$

The general mathematical model for the suspension system with a MR damper is:

$$\begin{aligned} M\ddot{q} + Kq &= B_f u + B_r z_r \\ M &= \begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}, K = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \\ B_r &= \begin{bmatrix} 0 & k_t \end{bmatrix}^T, B_f = \begin{bmatrix} -1 & 1 \end{bmatrix}^T, \; q = \begin{bmatrix} z_s & z_u \end{bmatrix} \end{aligned}$$

where m_s, m_u, k_s, k_t , z_s, z_u and z_r represent the same parameters and variables shown in the passive suspension system. The actuator force u is replaced by the damping force of a Magnetoreological damper F_{MR}.

3.4 Magnetorheological damper model

The MR fluids are smart materials that respond well to an applied magnetic field, modifying their rheological properties like viscosity and stiffness. The viscosity and stiffness changes are continuous and reversible, which makes possible the application of MR dampers for vibration absorption (Spencer *et al.* [Spencer1997]). The passive nature of the MR dampers limits their practical use to semiactive control, although this is sufficient to improve the rotor-bearing system response and extend the stability thresholds. In this work the Choi-Lee-Park polynomial model for the MR dampers is considered (see Choi *et al.* [Choi2001a]). We used a MR damper RD-1097-01, manufactured by *Lord Corporation*^{TM} (Silva & Cabrera [Silva2007]), whose damping force is:

$$\begin{aligned} F_{MR}(\dot{v}, I) &= \sum_{i=0}^{n=2}(\hat{b}_i + \hat{c}_i I)\dot{v}^i \\ v &= z_s - z_u \end{aligned} \tag{3.4.1}$$

where vertical displacements of the vehicle suspension is $v{=}z_s{-}z_u$, piston velocity for the MR damper is expressed as $\dot{v}{=}\dot{z}_s - \dot{z}_u$, the acceleration $\ddot{v}{=}\ddot{z}_s - \ddot{z}_u$ and the current control input I. The general coefficients ($\hat{b}_i$, $\hat{c}_i$), with respect to the positive acceleration (b_i^+, c_i^+) and negative acceleration (b_i^-, c_i^-) of the piston, are given by:

$$\hat{c}_i = \frac{(c_i^+ + c_i^-) + |c_i^+ - c_i^-|sign(\ddot{v})}{2}, \; \hat{b}_i = \frac{(b_i^+ + b_i^-) + |b_i^+ - b_i^-|sign(\ddot{v})}{2}, \; i = 0, 1, 2. \tag{3.4.2}$$

The experimental coefficients for a polynomial of order $n = 2$ are given in Table I.

Table I. Polynomial coefficients b_i and c_i				
index	Positive acceleration $\ddot{v}$		Negative acceleration $\ddot{v}$	
i	b_i^+	c_i^+	b_i^-	c_i^-
0	0.403	2.928	0.5426	−3.105
1	−18.3	1156	−18.549	1161
2	19.01	−561.3	8.6212	−372.5

3.5 Positive velocity feedback control

The Positive Velocity Feedback (PVF) control scheme is applied to attenuate the vertical movement in the suspension. This control is well-known in the literature as a modal control method for vibration attenuation, which was proposed by Goh-Caughey [Goh1985] (see also Inman [Inman2006a]). The scheme adds an additional DOF to the original system, considered as a virtual passive absorber or as a second order filter. The parameters of this controller can be selected using purely experimental data, which in fact makes the PVF technique very popular among structural and control applications. The positive velocity terminology comes from the fact that the velocity coordinate of the structure equation is positively fed to the filter, and the velocity coordinate of the compensator equation is positively feedback to the structure (Inman [Inman2006a]).

The synthesis of a PVF control for a multiple DOF system results in a closed-loop system given by:

$$M\ddot{q} + C\dot{q} + Kq = B_f u \tag{3.5.1}$$

$$\ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta = g\omega_f B_f^T \dot{q} \tag{3.5.2}$$

$$u = g\omega_f\dot{\eta} \tag{3.5.3}$$

In this case the control parameters are given by a gain g and filter constants ζ_f and ω_f. For this control scheme the stiffness K and mass M matrices are both symmetric and positive definite and, therefore, the closed-loop matrices $\hat{M}$ and $\hat{K}$ are symmetric and positive definite. The PVF affects only the closed-loop damping matrix,

$$\hat{C} = \begin{bmatrix} C & -g\omega_f B_f \\ -g\omega_f B_f^T & 2\zeta_f\omega_f \end{bmatrix}$$

Now, note that,

$$q^T \hat{C} q = \begin{bmatrix} q_1^T & q_2^T \end{bmatrix} \begin{bmatrix} C & -g\omega_f B_f \\ -g\omega_f B_f^T & 2\zeta_f \omega_f \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$
$$= q_1^T C q_1 - g\omega_f q_1^T B_f q_2 - g\omega_f q_2^T B_f^T q_1 + 2\zeta_f \omega_f q_2^T q_2$$
$$= q_1^T \left(C - \frac{g^2 \omega_f}{2\zeta_f} B_f B_f^T \right) q_1 + \frac{\omega_f}{2\zeta_f} \left(g B_f^T q_1 - 2\zeta_f q_2 \right)^T \left(g B_f^T q_1 - 2\zeta_f q_2 \right)$$

Because the second term is always non-negative then, to guarantee the closed-loop asymptotic stability the damping matrix $\hat{C}$ must be positive definite, which can be achieved if the gain g and ω_f are selected such that the matrix $C - \frac{g^2\omega_f}{2\zeta_f} B_f B_f^T$ be positive definite.

3.6 Semiactive Vibration Absorption

For control purposes it is considered the so-called inverse model for the MR damping force in (3.4.1), such that the current control input can be obtained as (see Choi *et al.* [Choi2001a]):

$$I = \frac{u - \sum_{i=0}^{n=2} \hat{b}_i \dot{v}^i}{\sum_{i=0}^{n=2} \hat{c}_i \dot{v}^i}, \quad \dot{v} = \dot{z}_s - \dot{z}_u \tag{3.6.1}$$

where u denotes the calculated control force.

It is important to remark that, in general, there is no singularity in (3.6.1) because $\hat{c}_0 \neq 0$ and the actual acceleration and velocity ranges are far from any singularity. For the application, we consider the quarter vehicle suspension system (3.3.1), which is controllable from the independent MR damping force F_{MR}. In Fig. 3.2 is described the control model.

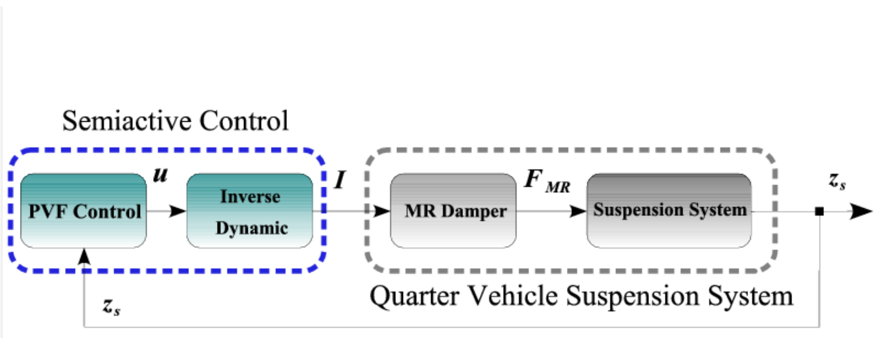

Figura 3.2: Schematic diagram of the semiactive control scheme.

3.7 Simulation results

The simulation results were obtained by means of MATLAB/Simulink, with the Runge-Kutta numerical method and a fixed integration step of 1 ms. The numerical values of quarter car model parameters are as the followings: $m_s = 44\,[kg]$, $m_u = 11\,[kg]$, $k_s = 3580\,[\frac{N}{m}]$, $c_s = 200\,[\frac{Ns}{m}]$ and $k_t = 33158\,[\frac{N}{m}]$.

In this simulation study, the typical road disturbance is shown in Fig 3.3 and set in the form of: $(a\frac{1-\cos(8\pi t)}{2})$ if $0.5 \leq t \leq 0.75$ and $3.0 \leq t \leq 3.25$ with $a = 0.08\,[m]$, or 0 otherwize.

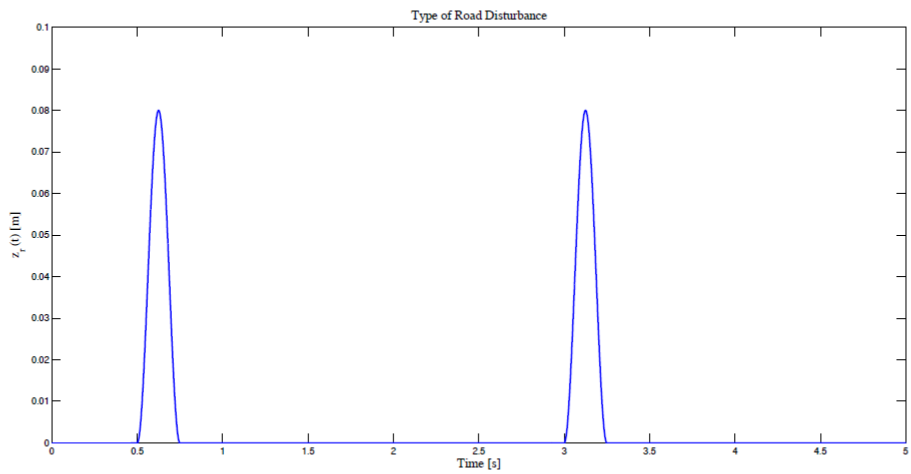

Figura 3.3: Type of road disturbance.

The application of a PVF scheme for the model of a quarter vehicle suspension system, has parameters as: $\omega_f = 10\,[\frac{rad}{s}]$, $g = 200$, $\zeta = 10$. In the Fig. 3.4 is shown the vertical displacements z_s and defletion $z_d = z_s - z_u$. The passive system has vibration amplitude very remarkable, using the MR damper F_{MR} and the control algorithm allow to reduce the body movement z_s up to 65%. The control current I is saturated to $0.6\,[A]$, leading to stabilized and attenuated responses with damping force about $400\,[N]$ (see Fig 3.5).

3.8 Conclusions

In this work a 2 DOF model for a suspension system with a MR damper is addressed. The mathematical model of a quarter vehicle suspension system and magnetorheological damper model reflect a similar behavior in an experimental platform, therefore the presented work helps to predict the dynamic behavior and to study some control algorithms. The MR suspension allows the synthesis of semiactive control schemes through current control inputs, driving correspond-

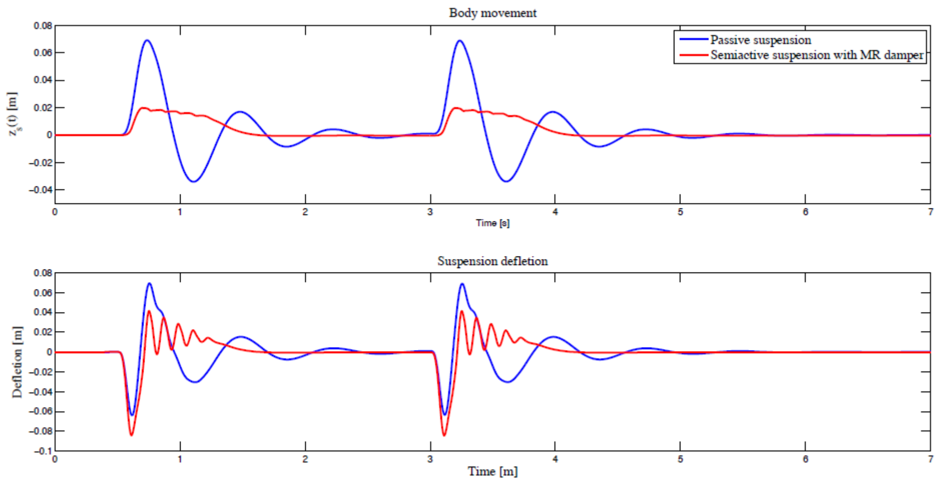

Figura 3.4: Simulation results of the passive suspension ($F_c = c_s(\dot{z}_s - \dot{z}_u)$), and using a MR damper ($F_{MR}(\dot{v}, I)$).

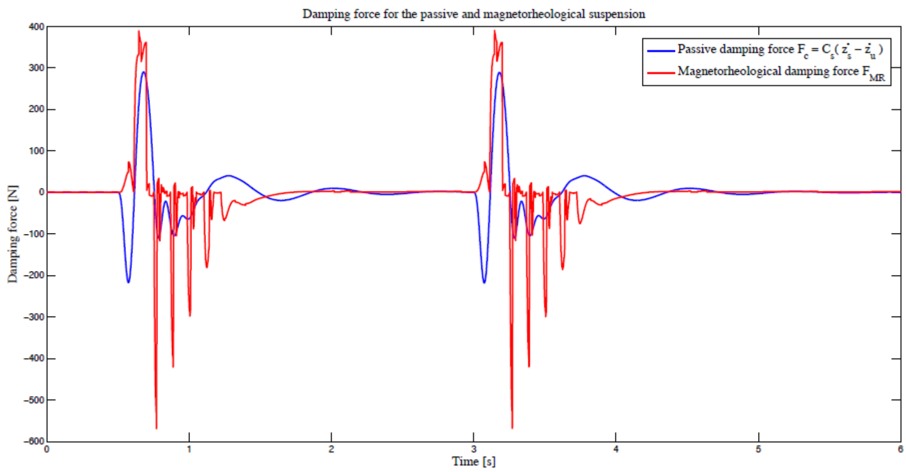

Figura 3.5: Damping force for passive suspension ($F_c = c_s(\dot{z}_s - \dot{z}_u)$), and the suspension with MR damper ($F_{MR}(\dot{v}, I)$).

ing damping force, with the use of PVF controller. It only one measurement is needed, this is, vertical velocity for chassis. The PVF scheme reduces the vibration amplitude up to 65%, with respect to the passive suspension. Further work is currently completed to get experimental validation in a physical platform.

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