Capítulo 16

Mathematical Models in Geogebra for Teach, Learn, and Research

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Abstract: There are today many software packages which allow people to perform calculations and geometrical and visual representations for many models. These packages, being quite powerful, are also expensive, and not always affordable for students or researchers. There are, although, free alternatives like geogebra, a dynamic mathematics software. On this paper there are shown some geogebra uses for teaching, learning and researching, such as reflective surfaces simulation, segmented solar collectors, caustics, evolutes and mechanical phase space calculations. These examples show the fact it is possible to build models for real physical systems for learning, teaching and researching.

Keywords:dynamic mathematics, mathematics modeling.

Resumen: Existen en la actualidad numerosos paquetes de programas que permiten realizar cálculos y representaciones geométricas y visuales de diversos modelos. Estos paquetes, si bien suelen ser bastante poderosos, también son caros, y no siempre están al alcance de los estudiantes o investigadores. Existen, sin embargo, alternativas libres como Geogebra, un programa para geometría dinámica. En el presente trabajo se presentan ejemplos del uso de Geogebra en procesos de enseñanza-aprendizaje e investigación tales como la simulación de superficies reflectoras, colectores solares segmentados, el cálculo de cáusticas, evolutas y espacio fase de sistemas mecánicos. Estos ejemplos muestran que es posible realizar modelos de sistemas físicos

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reales para procesos de enseñanza-aprendizaje o investigación.

Palabras clave: matemáticas dinámicas, modelamiento matemático.

16.1 Introduction

The term mathematical model leads to understand two ideas. First one is related with a scientific model based on a mathematical formulation in order to express items or objects and relations among them, along with certain restrictions or qualities. Second one is related with math philosophy, where formal models for math are proposed. It is the first notion the used one for this paper.

Applied mathematics is another important concept for this paper, and a very broad definition is the one given by Tim Pedley[Wilson2011]: "Applying mathematics means using a mathematical technique to derive an answer to a question posed from outside mathematics". Without discussing why mathematics is applicable this paper focuses on just one field and its applicability: analytic geometry.

Analytic geometry has two fundamental problems, each one being the inverse of the other, constituting the fundamental problem of analytic geometry. First problem deals with a geometric interpretation of an equation. Second one deals with finding an equation which describes a set of points.[Lehmann1980] Solving these two problems for some cases like points, straight lines, circumferences, and relations among them had proven to be a powerful tool for describing systems found in nature, like the ones studied by classical mechanics.

When people describe simple systems like a free point body falling free, or pendulum dynamics not only a set of points describing trajectories are of interest, but also forces and interactions can be described by means of analytic geometry tools, and the same can be said regarding some optical systems.
From some years to now computers are clearly part of all aspects of our lives. We work with computers, communicate through computers, entertain with computers, teach with computers, and do research with computers. A set of digital competences are required to perform these days, and academic life is not the exception: there exists computer algebra systems, visualization software, simulation software, and of course dynamic mathematics software. These software packages could be commercial ones, which use to be expensive, or free ones, which use to be open source, but not all. There is a special case: geogebra, a dynamic mathematics package.

16.2 Geogebra

As said before, Geogebra is a dynamic mathematics package, although some days before writing this paper it was claimed to be a dynamic geometry system. As the authors claim[GeogebraWeb] “Geogebra is a multi-platform mathematics software that gives everyone the chance to experience the extraordinary insights that math makes possible”. Also the authors address their software for students, teachers and schools. Students because “it makes math tangible”, teachers because “allows teachers to continue teaching”, and students because “... get better results”, but they forgot researchers!
For most students, mainly before university, math is a nightmare, a series of facts to be memorized and then regurgitated. People barely see or feel a connection between mathematics, their life or the world. When teachers give their lectures sometimes diagrams, plots, data tables and equations are related, and most of the time students don’t see the connection. Even more, when something changes, it could be hard to derive or follow consequences. Geogebra gives students the possibility to see, touch and experience math in new ways.
For teachers, geogebra gives the opportunity to extend the lecture by means of interactive materials which the students can use and play with in order to learn properties or concepts, also for preparing diagrams or presentations, and of course to understand and experiment in different ways with theorems, mathematical models or ideas.
And finally, for researchers, it gives a free dynamical graphical tool for dynamic mathematical modeling. It is true the fact there are commercial and freely available software packages, even compilers for writing programs which solve complex problems, and I think Geogebra can’t substitute them for some tasks, but as it will be shown, it is possible to make geometrical models for physical systems allowing decision making during design processes, diagram preparation and fasts results for sketches, thinking about geogebra as a substitute for model hand drawing.
People may say “If students use Geogebra for problem solving, then a mathematical functional analphabetism could arise, just the same as square root algorithm and calculators”. Well, it may be true, but the same is for other packages such as Mathematica, Mapple, Matlab and ViewLab for the commercial ones or maxima, octave, gnuplot among others on the free side. It could be not far from reality if it is heard about researchers who depends on some software package for equation solving, and if, let us say, Mathematica is unable to solve the algebraic equations, then the researcher is unable to finish his work. Please note this is very different from studies on some areas where long or complex simulations are used, like the ones in chaos theory.
Geogebra has several tools for geometry, calculus, algebra, spreadsheet, graphing and statistics with connection and full dynamics for geometry, algebra

and spreadsheet. In the next section some tools will be presented in order to explain how to build models for some physical systems.

16.3 Building mathematical models for some physical systems.

On some of the next examples a geogebra screen is shown with numbers allowing easy identification of tools and characters for geometrical objects. On the text explaining the example, numbers and characters in parenthesis match numbers and characters in the figure for the paragraph.

16.3.1 Reflecting surfaces

Basic study of geometrical optics could be started with learning and experimenting how light reflects on smooth plane surfaces. The law of reflection for plane surfaces states: "the angle that the reflected ray makes with the normal at the point of incidence is always equal to the angle the incident ray makes with the same normal. Note carefully that the incident ray, reflected ray and normal always lie in the same plane"[Pedrotti].

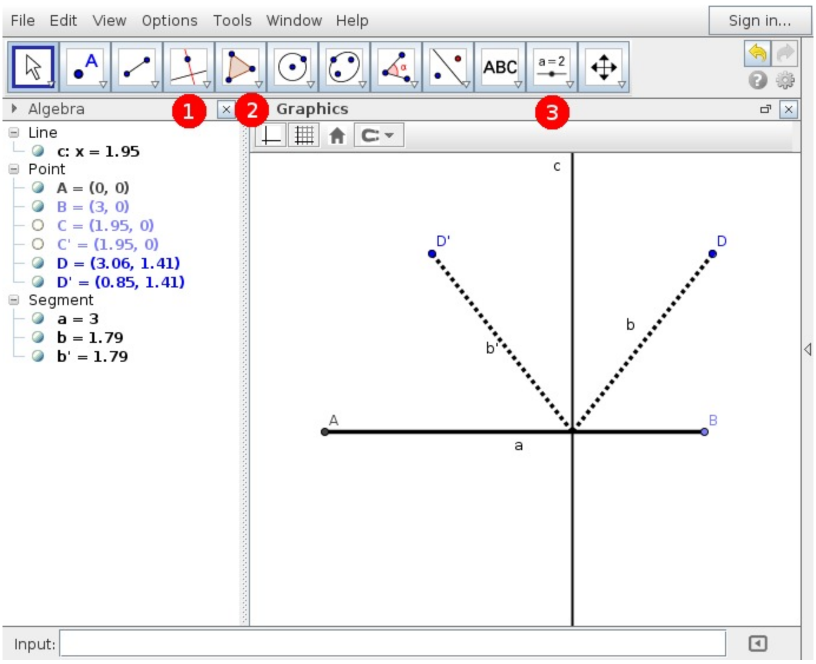

Figura 16.1: Geogebra construction for law of reflection for plane surfaces.

For this example (figure 16.1), a "line segment"(1) representing the plane sur-

face(a) will be used. Then, choosing a point on the surface another line segment will be constructed, representing the incident ray(b). With the "perpendicular line" tool(2) the normal will be shown(c), and then with the "reflect about line"(3) tool the reflected (b') ray is found. This example gives the basic tool for modeling caustics.
Second example is caustic calculation. A caustic is the set of points where two reflected rays intersects[Castañeda2012]. In order to use the previous example the surface is modeled as made of several small reflecting plane surfaces. In this way, if the expression representing the surface is smooth enough the mean value theorem can be used to select two contiguous segments and then reflect rays coming from any kind of light source. On figure 16.2 a smooth surface is introduced writing its equation in the input bar, then a point A is chosen. Using the point A a circle using "circle with center and radius" is drawn, and the two intersections with the surface are used with the center to build two line segments. Then using "midpoint or center" tool the reflection points on plane surfaces are found.

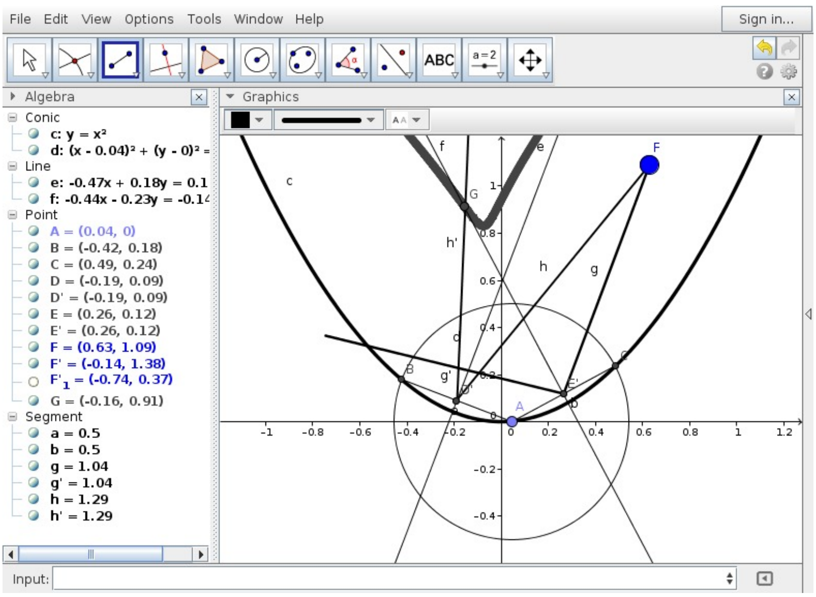

Figura 16.2: Caustic construction using the mean value theorem for smooth reflecting surfaces.

As before, perpendicular lines are drawn, and then the incident rays. After this, a point is selected as a point light source (this is done for simplicity, using vector tools it is possible to simulate parallel rays), then the rays reflected as before. Final step is to intersect both reflected rays, and select "trace on" with mouse right button over the intersection point. After this, just move the center of the circle along the curve and the caustic is found. Just two caveats. First is regarding incident and reflected rays, it is better to use "ray" option on the

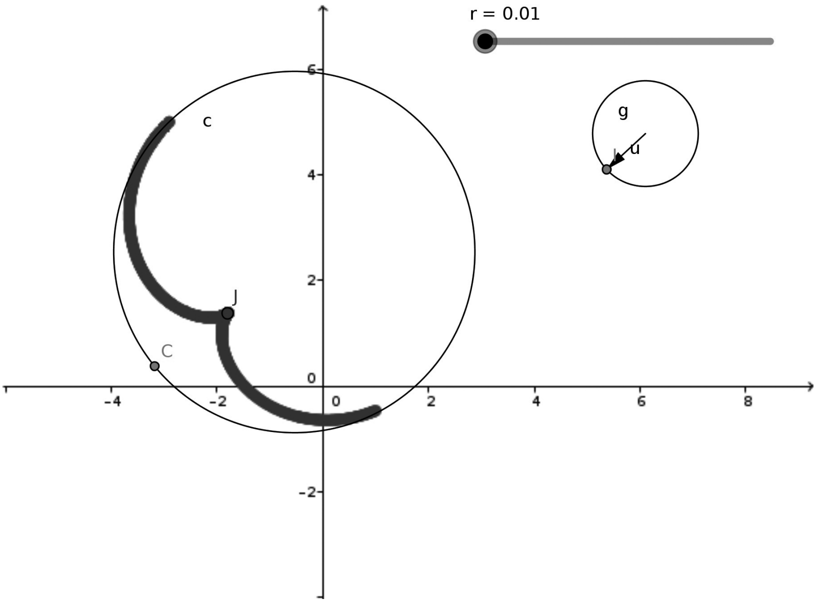

Figura 16.3: Caustic of the circle calculated with the method used in figure 7.2. Radius is 0.01 arbitrary units and light rays are in the direction shown by the vector on the right side.

tool bar in order to make rays enough large for some systems. Second is a too cluttered screen. It it possible to hide some elements, in this case the circle, plane segments, normals are good candidates for hiding, but are left now for the sake of understanding. Just one more comment, this example is not good in the sense that the circle radius is too large, the best option is to use the slider tool to change radius as desired. Starting with a large one, after building the model, a small radius is chosen then the resulting caustic is very close to the real one. On figure 16.3 the caustic for the circle and parallel rays with direction vector are shown. Auxiliary elements are hidden.

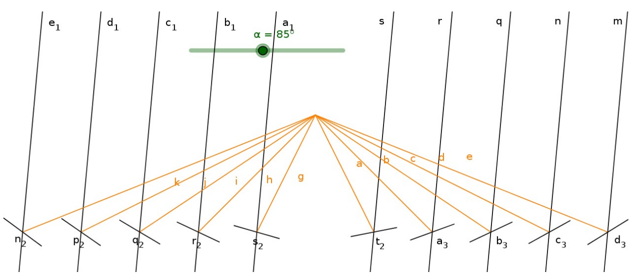

Figura 16.4: Simple simulation showing a segmented solar collector. Parallel rays coming from the Sun are shown, reflected rays converge to the same points. As alpha angle is changed, each surface rotates.

In order to finish optical systems examples, only a picture of a segmented solar collector is shown. The same reflection law is used to construct several planar mirrors reflecting parallel sun rays and making them to converge on the same point. For simplicity only the center of each mirror is used for calculations. This system is being constructed as a research project from the author, who also wrote optical software simulation which is used for a detailed caustic calculation on this example whose results are yet to be published but first ones are on [Castañeda2012]. Just as stated before, these geogebra constructions purpose are for teaching, showcasing or experimenting for ideas, and in several cases never will substitute complex and detailed simulations.

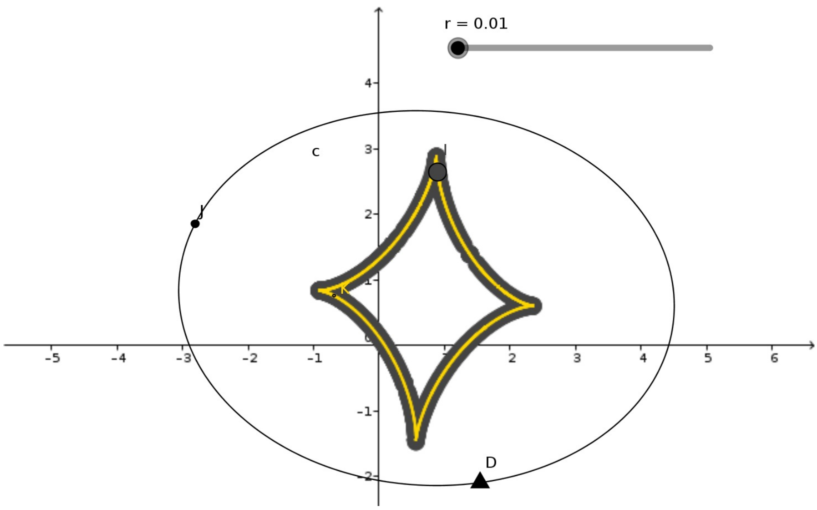

Figura 16.5: Building curvature by two methods.

16.3.2 Evolute

Going onto another topic, Geogebra also has calculus capabilities. Using derivatives some algorithms like Newton-Raphson could be teached. Geogebra actually has a "Curvature" and "VectorCurvature" command. Using "Curve" command it is possible to build parametric curves, then use the same recipe as before for tangent segments using the mean value theorem, after that, tracing perpendicular lines and finding their intersection a point candidate for center of curvature is found. Of course making the circle radius small enough, moving the center along the curve and tracing the point the evolute may be found, but this method sometimes fail. For this example the "Ellipse" tool is used for the curve, and the rest as before. Results using this method and "Curvature" command are shown in figure 16.5.

16.3.3 Phase space

The last example will be that of the pendulum phase space. For this example pendulum's Hamilton equations will be used inside a script.
The Hamilton's equations is a first order differential system of equations, and can be solved numerically. For this example the Euler's method will be use. The main idea is to evaluate generalized momentum and coordinate at a given initial condition and then move onto the next point using a small amount of time dt. In geogebra terms this means some input boxes have to be used for initial conditions and a slider with animation option turned on.

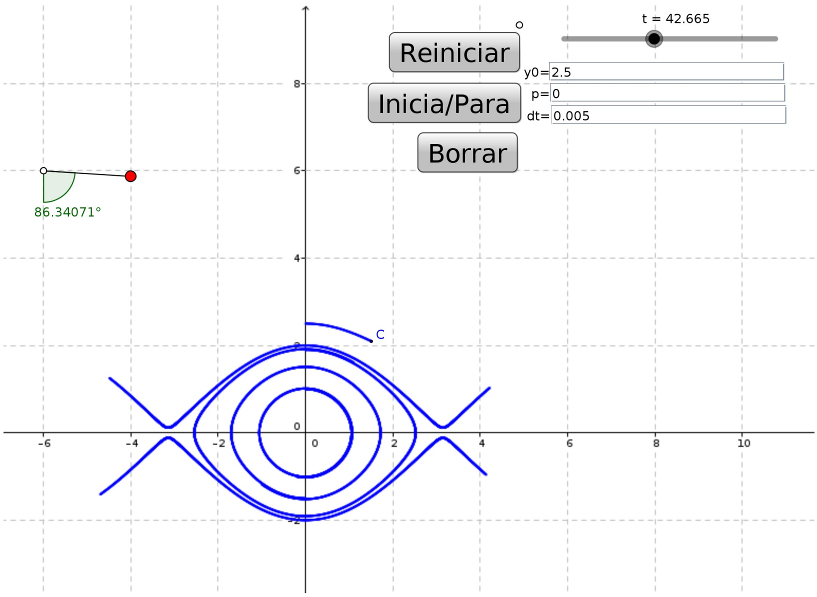

Figura 16.6: Pendulum phase space

Geogebra objects can use scripts, and some others are used as "animation engines". In this case a script is shown for the slider associate with time. Also some buttons are used to control animation and erase the screen, although moving coordinates system erase object traces. Also there is the problem of Euler's method, is not the best for integration, but is easily scriptable on systems like these.

The final result for phase space simulation is shown in figure 16.6. where also the pendulum being simulated is shown. On figure 16.7 the slider script is shown. This is great for teaching some concepts regarding Hamiltonian Mechanics, chaos theory, basic simulation and numerical methods.

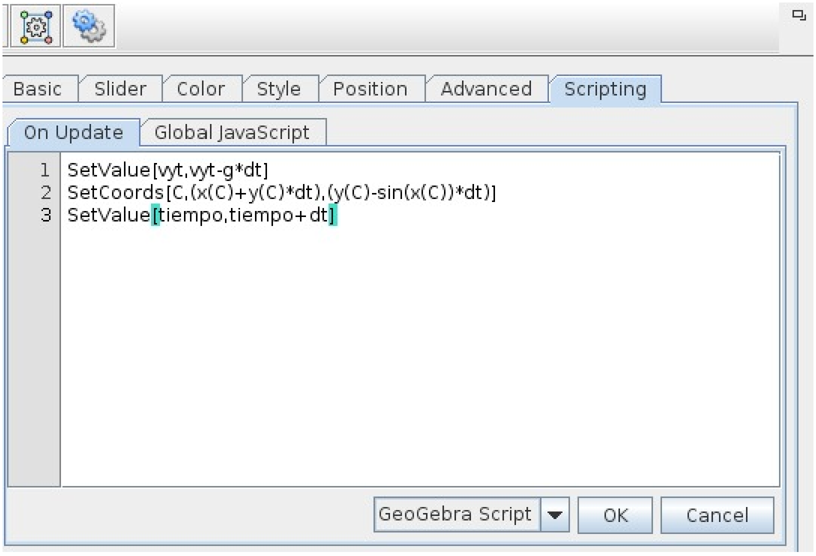

Figura 16.7: Slider script for Euler's integration method.

16.4 Conclusions

The presented examples show how Geogebra can be used to build mathematical models for representing some systems. Particularly the segmented solar collector is a real example of a research application, where a sketch representing the real system can be prepared before building a full simulation program. These applications can be done with other software packages, but Geogebra presents an intuitive yet powerful interface.

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